British Fertility Heads South: Understanding the recent decline

John Ermisch
Leverhulme Centre for Demographic Science and Nuffield College
University of Oxford

18 May 2021

Abstract

BACKGROUND

Fertility in Great Britain has fallen considerably during the past decade. The total fertility rate reached its historically lowest level in 2020.

OBJECTIVE

To improve our understanding of the decline in British fertility by using data on individual women during 2009-2020 from *Understanding Society*, which is a panel survey of the members of approximately 40,000 households.

METHODS

Estimation of a model of age and parity-specific birth rates including year-effects on individual data and cross-validation of it with external sources from registration data. Translation of the parameter estimates into more easily interpreted concepts such as period parity progression ratios and the total fertility rate and computation of their standard errors.

RESULTS

The decline in first birth rates appears to be primarily responsible for the decline in the TFR during the past decade, and women whose education is below degree level experienced a larger fertility decline.

CONCLUSIONS

Either Britain is embarking on a regime of a high level of childlessness not seen since that among women born in the early 1920s, or we can expect a recovery in period fertility in the future.

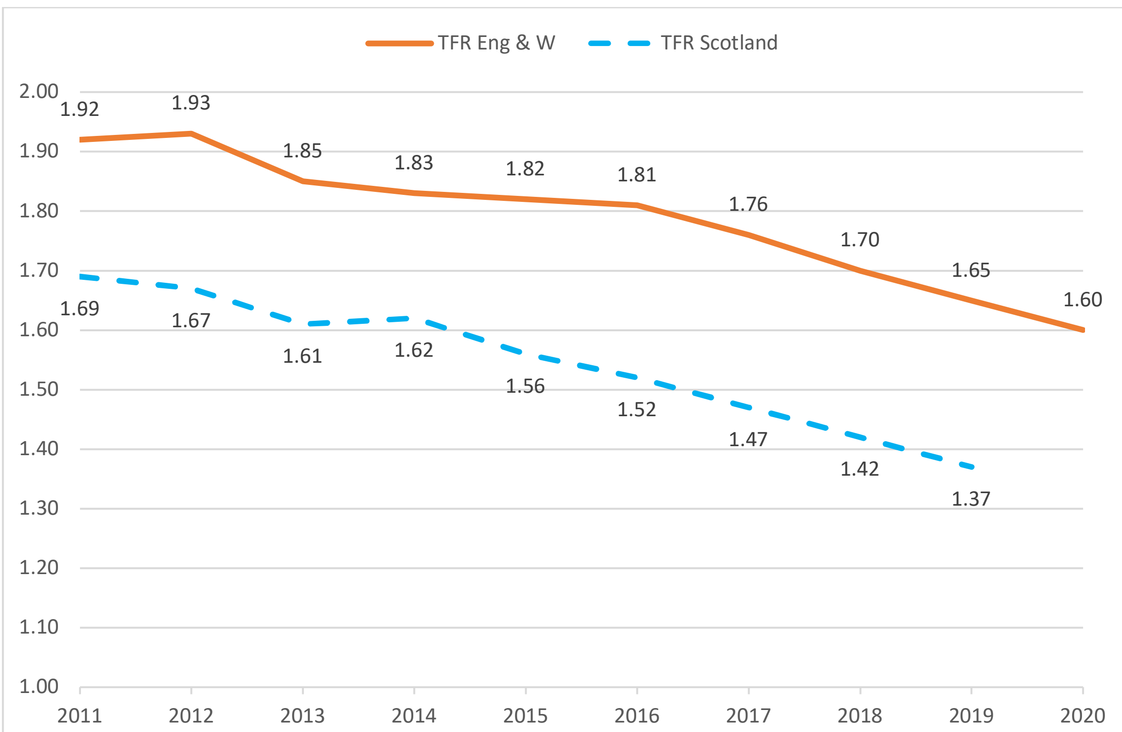
CONTRIBUTION

Use of individual level panel data to estimate a model of parity-specific birth rates, which is cross-validated against registration data, and used it to provide insights into what lies behind the recent decline in British fertility.

1. *Introduction: trends in British fertility*

During the past decade, fertility in England and Wales has fallen considerably, particularly since 2016. As illustrated in Figure 1, the Total Fertility Rate (TFR)[1] fell by 0.3 children per woman, reaching 1.6 in 2020, equal to the current level in Germany, although still above that in Italy (1.3).[2] There has also been a fall in Scotland, particularly since 2014, and its level is closer to the Italian one. The paper focuses on England and Wales, which produced 93% of births in Great Britain in 2019.

Figure 1: Total Fertility Rate (children per woman), England and Wales and Scotland



---

[1] The TFR is the number of children a woman would have over her reproductive life if she experienced the age-specific fertility rates prevailing in a particular year.

[2] The 2020 fertility rates have been adjusted to compensate for having only three quarters of data. No data has been released for subsequent months for reasons related to the pandemic.

The aim of the paper is to obtain a better understanding of recent changes in fertility and what they may imply for future developments. Its main contribution is to use individual level panel data over the last decade from the UK Household Longitudinal Study to estimate a model of parity-specific birth rates, to cross-validate the model against registration data and to use it to provide insights into what lies behind the recent decline in British fertility, particularly its parity composition and education differentials.

Figures 2 and 3 provide a longer perspective. Period fertility has never been as low as it was in 2020. There have of course been declines and recoveries before. For instance, between 1990 and 2001 the TFR fell from 1.84 to 1.63 and then recovered to 1.94 by 2011. This is because period fertility rates like the TFR and the General Fertility Rate (GFR: births per 1,000 women aged 15-44) reflect changes in timing as well as any change in completed family size. Figure 3 shows completed fertility by birth cohort along with the TFR 25 years after the cohort's birth. Cohort fertility since the 1920 birth cohort exhibits one wave, peaking at 2.42 for the 1934 cohort, and is on a downward trend since then, reaching 1.92 for the 1974 cohort, who have reached the end of their reproductive years in 2020.[3]

[3] The cohort data are from:

https://www.ons.gov.uk/peoplepopulationandcommunity/birthsdeathsandmarriages/conceptionandfertilityrates/bulletins/childbearingforwomenbornindifferentyearsenglandandwales/2019

Figure 2: General Fertility Rate (per 1,000 women aged 15-44) and Total Fertility Rate (per woman), England and Wales, 1938-2020

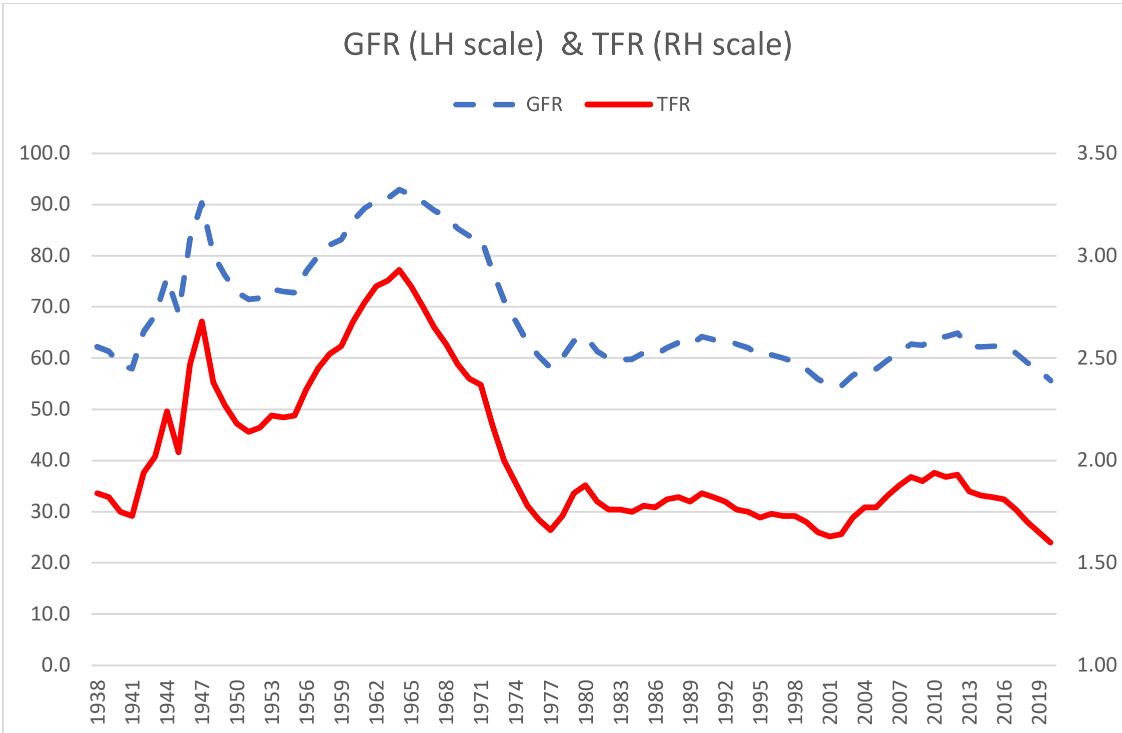


Figure 3: Cohort Fertility and TFR 25 years later by birth cohort

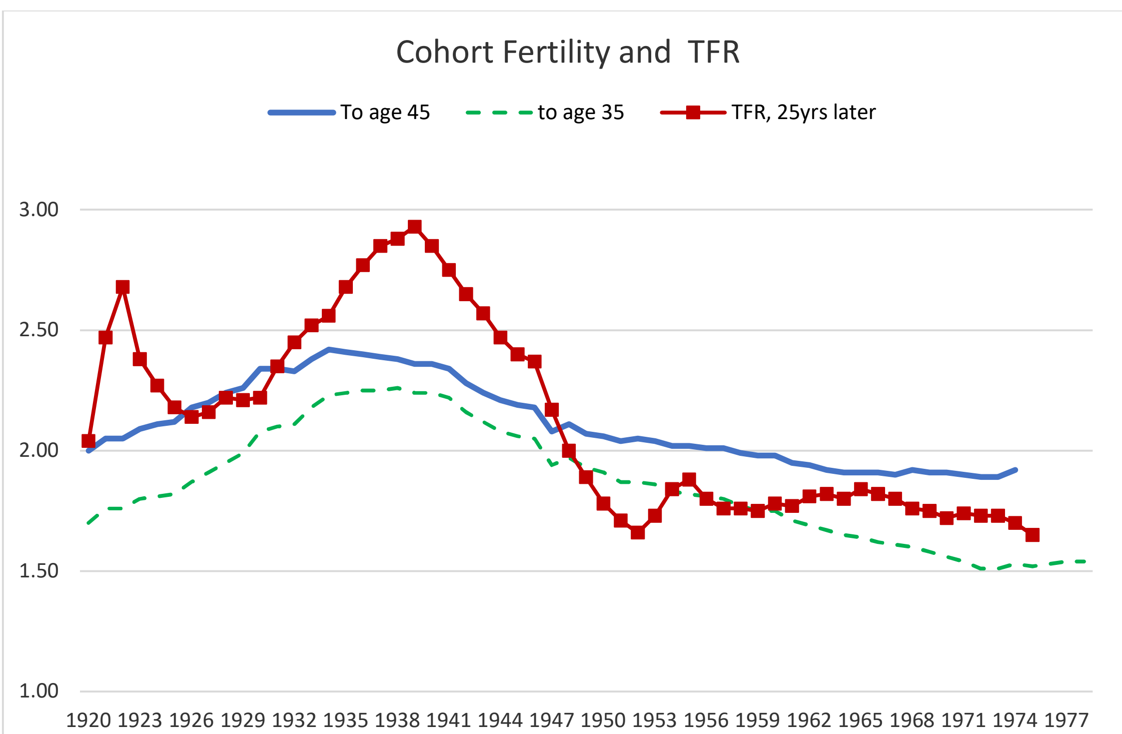
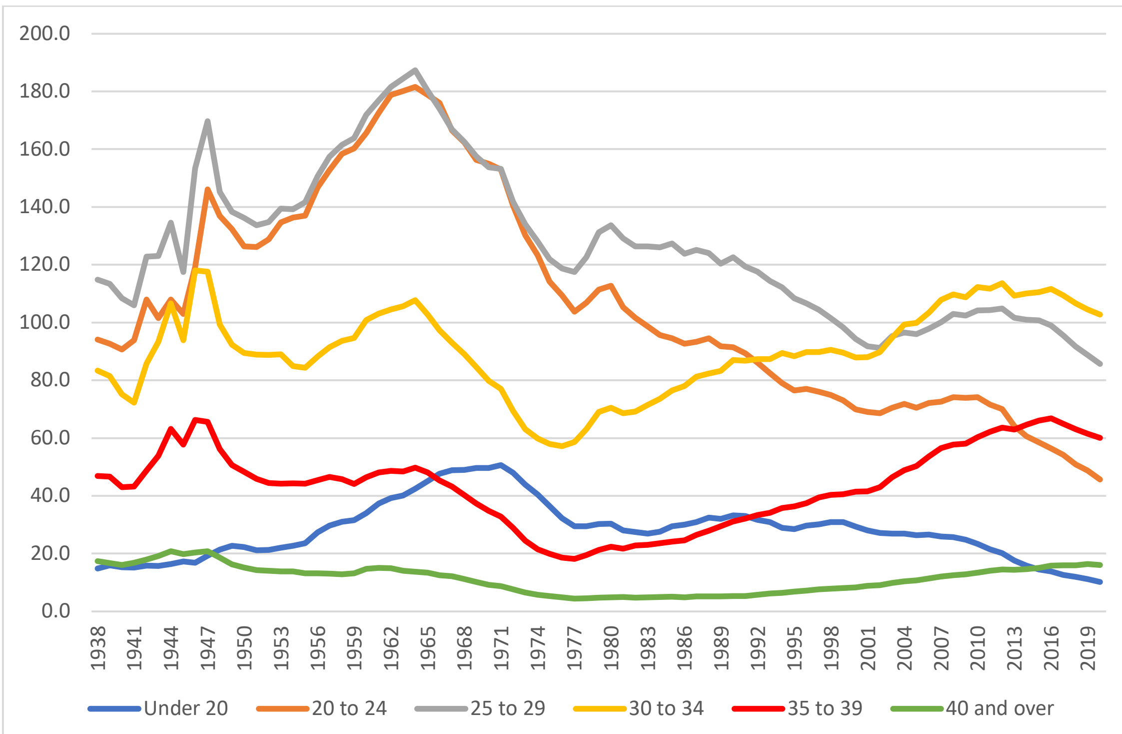


Figure 4 provides information about age-specific period fertility rates. Long term declines since the mid-1960s in the rates for women aged under 30 are evident, accompanied by a rise among older women, particularly those aged 30-39, producing a later average age at motherhood. During the past decade the standardised mean age of the mother at childbirth continued its secular increase (beginning in 1975), rising from 29.5 in 2010 to 30.7 in 2019. There has, however, been a decline since 2016 in rates for all age groups under 40.

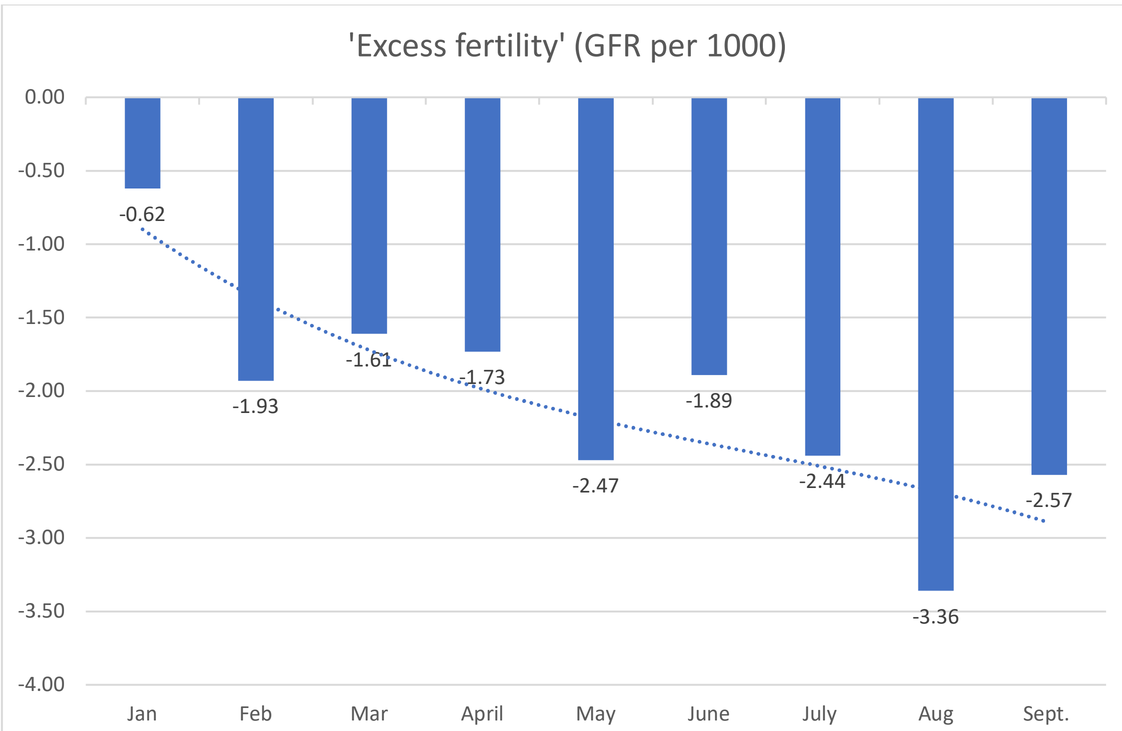
Figure 4: Age-specific fertility rates (per 1,000 women), England and Wales, 1938-2020



The correlation between the GFR and the TFR is high (0.987 over the period in Figure 2), making it a useful guide for how the TFR evolved during the past year. Figure 5 reports for 2020 the monthly GFR adjusted for the average fertility in each calendar month during 2011-

2019,[4] which is labelled 'excess fertility', along with a fitted (polynomial) trend line. It suggests a continuing decline in fertility during 2020 (the latest figures are for September). Birth registration data do not provide information on birth rates by birth order in recent years nor for education groups. The remainder of the paper fills this gap by using fertility data from individual women from longitudinal data during the past decade.

Figure 5: 'Excess GFR' during 2020, England and Wales



2. *Variation in fertility among women in Great Britain 2010-2020*

Estimation of parity-specific birth rates provides insights into fertility trends during the past decade. The source of information for the analysis is *Understanding Society*, also known as

[4] Monthly adjustment based on a monthly GFR model with fixed month and year effects for the period 2011-2019.

the UK Household Longitudinal Study, which is a longitudinal survey of the members of approximately 40,000 households (at Wave 1 during 2009-11) in the United Kingdom. Households recruited at the first round of data collection are visited each year to collect information on changes to their household and individual circumstances. Annual interviews are carried out face-to-face in respondents' homes by trained interviewers. The analysis used data on women born since 1970 during the first ten waves (the tenth collected during 2018-2020). Fertility was measured by births between waves of the panel survey.

2.1 *Method*

The main statistical method is the estimation of parity-specific functions for the annual birth probability. Each probability was assumed to depend on age, time since the last birth (other than parity zero) and interview year. The three parity-specific equations estimated for parities zero, one and two and above take the following form:

$$\ln\left(\frac{p_{itk}}{1-p_{itk}}\right) = \alpha_{0k} + \alpha_{1k}age + \alpha_{2k}age^2 + \delta_{1k}duration + \delta_{2k}duration^2 + \gamma Scotland \sum_{j=2010}^{2020} \mu_{jk}, \qquad k = 0, 1, 2 \textit{ or more}.$$

where p_{itk} is the probability of woman *i* at risk for a birth of parity *k* having a birth between waves *t-1* and *t*; duration is the years since the last birth for parities above zero (with δ_{10} and δ_{20} set to zero); *age* is the woman's age in years; $Scotland$ is a binary variable indicating residence in Scotland; and μ_{jk} are interview-year fixed effects. In the equation for parities two and above, the equation also contains $\sum_{k=2}^{5} \gamma_k parity_k$, measuring baseline fertility for each parity.

The sample consists of up to 9 pairs of consecutive years during which a birth could occur for each woman. There are no 'off-the-shelf' weights to assure the representativeness of such a sample to compute estimates of population means such as birth rates, but the sample can be used to estimate the model parameters on the assumption that these are constant across women and over the decade of analysis. The parameter estimates are, therefore, based on unweighted data. This can, however, present a problem for the interpretation of the year-specific parameters μ_{jk}. They could reflect both sample composition effects and 'true' period influences.[5] Descriptive statistics are provided in Appendix Table B1.

2.2 *Model parameter estimates*

Overall, we observe 4,586 births during the period from 71,078 woman-year observations (from 15,536 women contributing between 1 and 9 waves of observation) of which 1,743 and 1,698 births are from parities zero or one, respectively, and 1,145 births are at higher parities. The estimates of average marginal effects from the model are shown in Table 1. Parameters associated with age and duration are precisely estimated. These parameters reflect the processes of partnering (particularly for first births) and of partnership dissolution as well as births within partnerships. For second and higher order births, the impacts of age and duration are hard to interpret on their own because age advances with duration and higher parities are achieved at higher ages. As an example of their operation in combination, consider a woman having her first child at age 30. The model predicts that the probability of a second birth peaks three years later (at a second birth rate of 0.21) and then declines. The model's second birth age *cum* duration profile implies that, at 2010 birth rates, 84% of women having a first birth at age 30 would eventually go on to have a second birth. Differences in the baseline parity-specific birth probability among the higher parities are not

[5] The birth may have occurred in the calendar year preceding the interview year, making interpretation of, for example, '2016' as '2015-16'.

precisely estimated. Scotland has lower second and higher order fertility rates, but there is considerable imprecision in their estimated effects.

The estimated year-effects are not precisely estimated for parities above zero, but for parity zero there is strong evidence for lower first birth rates since 2013. This suggests that the recent decline in the TFR may reflect further postponement of motherhood in recent years. But, as noted earlier, the year-effects may also reflect compositional changes in the women interviewed in individual years. To explore this issue further, the next section reports on a cross-validation of the model with annual birth registration data.

Table 1: Average Marginal Effects: Parity-specific Models with Age, Duration and Year effects (Robust Standard Error* in Parentheses)

Variable	**Parity 0**	**Parity 1**	**Parity 2+**
Age	0.043 (0.002)	0.056 (0.005)	0.014 (0.003)
Age-squared	-0.001 (0.0003)	-0.001 (0.00008)	-0.0003 (0.00004)
Duration	--	0.020 (0.003)	0.004 (0.001)
Duration squared	--	-0.003 (0.0004)	-0.001 (0.0001)
Parity: (ref=2):	--	--	
3	--	--	0.002 (0.003)
4	--	--	0.004 (0.005)
5	--	--	0.018 (0.010)
Scotland	0.004 (0.005)	-0.009 (0.011)	-0.007 (0.005)
Year (ref=2010):			
2011	-0.007 (0.008)	0.007 (0.015)	-0.005 (0.003)
2012	-0.001 (0.008)	0.011 (0.015)	0.001 (0.007)
2013	-0.016 (0.008)	0.024 (0.015)	0.001 (0.007)
2014	-0.023 (0.007)	0.011 (0.015)	-0.011 (0.007)
2015	-0.021 (0.008)	0.018 (0.016)	-0.001 (0.007)
2016	-0.024 (0.007)	0.006 (0.016)	-0.014* (0.007)
2017	-0.032 (0.007)	0.002 (0.016)	-0.004 (0.007)
2018	-0.026 (0.007)	0.001 (0.016)	-0.006 (0.007)
2019	-0.023 (0.008)	-0.021 (0.019)	0.003 (0.009)
2020	-0.027 (0.021)	0.104 (0.087)	-0.020 (0.020)
N women N woman-years	9,092 31,399	3,958 12,060	5,696 25,837

*Allowing for correlation in women's observations over time.

2.3 Cross-validation of model

The cross-validation exercise is designed to gauge the extent to which fertility behaviour captured in the *Understanding Society* data is consistent with the TFR data for England and Wales. It was carried out in the following way. First, the model in Table 1 was re-specified for parities higher than zero to a model with just age- and year-effects at each parity plus the Scottish-English difference (the parity zero model is the same as in Table 1).[6] Most importantly, estimates of this model produced similar year-effects to those in Table 1.

---

[6] Because *duration=age-age₀*, where *age₀* is when a woman enters the population at risk for that order birth, the model in Table 1 can be re-written as

$$\ln\left(\frac{p_{itk}}{1-p_{itk}}\right) = \alpha_{0k} + (\alpha_{1k}+\delta_{1k})age + (\alpha_{2k}+\delta_{2k})age^2 - \delta_{1k}age_0 + \delta_{2k}{age_0}^2 - 2\delta_{2k}age_0 age + \gamma Scotland \sum_{j=2010}^{2020} \mu_{jk}$$

In effect, the new model specification which drops *duration* and its squared value averages over the population at risk entry ages, so that

$$\ln\left(\frac{p_{itk}}{1-p_{itk}}\right) = \alpha_{0k} + (\alpha_{1k}+\delta_{1k})age + (\alpha_{2k}+\delta_{2k})age^2 - \delta_{1k}E(age_0) + \delta_{2k}E({age_0}^2) - 2\delta_{2k}E(age_0)age + \gamma Scotland \sum_{j=2010}^{2020} \mu_{jk}$$

The age slope in the new model depends on $-2\delta_{2k}E(age_0)$ and the intercept changes by $\delta_{2k}E({age_0}^2) - \delta_{1k}E(age_0)$. Using the Bayesian Information Criterion (BIC), the model with duration variables is preferable, but for third births there is little to choose between the two models (BIC=8611 cf. 8619); for second births the difference is larger (BIC=9165 cf. 9443). Having second and higher order births as a function of duration at risk for that birth

The sequence of parity-specific fertility transitions was simulated using the year-effects that apply in each year. For parities three and higher, the age-specific birth rate profiles were assumed to be the same as for parity two. The estimated transition rates imply period parity progression ratios for each birth order *j*: PPR_j.[7] Appendix A provides the details. From these the TFR is computed as:

$$TFR = PPR_1 + PPR_1 PPR_2 + PPR_1 PPR_2 PPR_3 + PPR_1 PPR_2 PPR_3 PPR_4 + \cdots$$

The predicted TFRs for each year are shown in Figure 6 as 'parity-specific model'. The correlation of predicted and actual is 0.77, and the root-mean-square error is 0.119.

Heckman and Walker (1990, p.1420) make the case that 'tests of the time series properties of an aggregated micro model offer evidence on the fit of a model in a metric other than the one used to estimate the model.' One test is whether the differences between the TFR predicted from the micro model and the actual TFR (e_t) are serially correlated. If they are, then the model is mis-specified. To carry out the test, as in Heckman and Walker (1990), the following regression is estimated: $e_t = \alpha + \rho e_{t-1} + u_t$, where u_t should be independently distributed over time ('white noise'). One then tests whether ρ is significantly different from zero. This mis-specification test supported the model (the p-value for the test that $\rho = 0$ is 0.175), and the intercept (α) was not significantly different from zero (p-value=0.93), indicating the model also predicts the level quite well. Also, the Ljung–Box (1978) Q test cannot reject the null hypothesis that the prediction errors (e_t) are independently distributed (a p-value of 0.098). Thus, the model performed well in replicating the TFR and its decline,

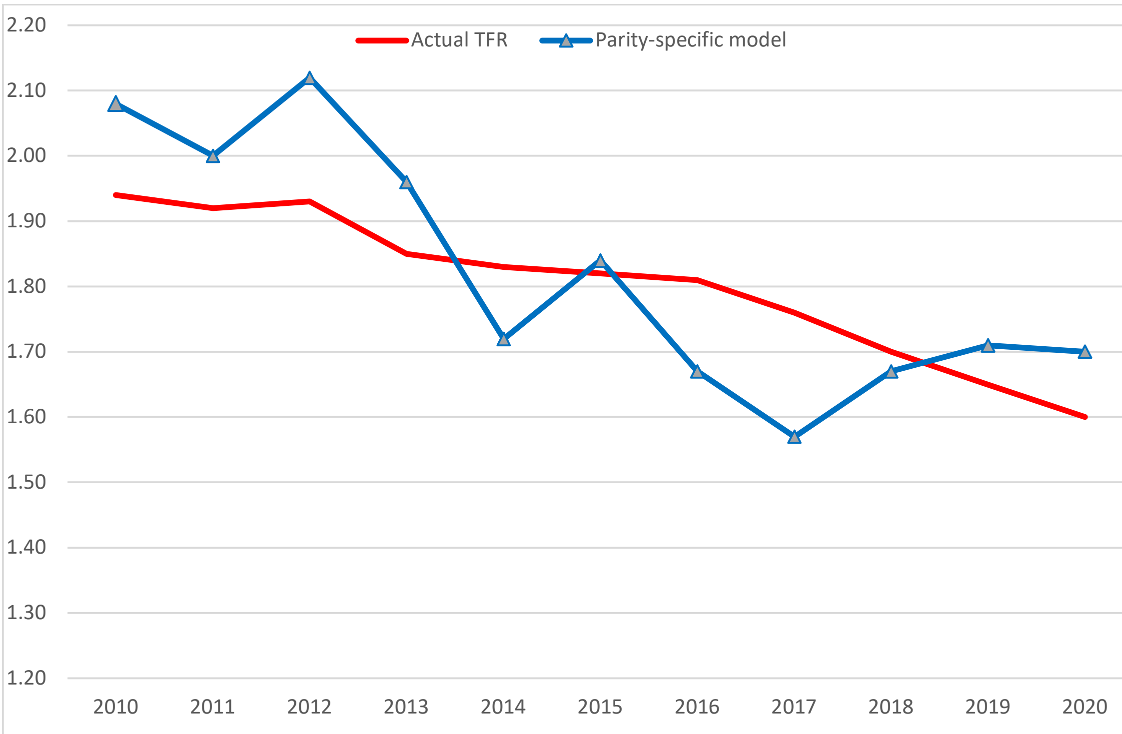
---

order complicates the bootstrapping simulations used to estimate the standard errors considerably.

[7] See, for example, Henry (1953), Feeney (1983) and Ni Bhrolchain (1987).

suggesting that the year-effects mainly reflect real changes in fertility behaviour over the 2010-20 decade, not just sample compositional effects.

Figure 6: TFR predicted by the demographic models and actual TFR



2.5 *Statistical inference*

Although it is more useful to think about the results from the fertility model in terms of parity progression ratios and the TFR than average marginal effects, we need some idea of the precision of the estimates of these quantities, namely their standard errors. These are not straightforward to calculate for the PPRs or the TFR other than via bootstrapping, as Appendix A explains. The estimates of the individual year-effects were not precisely estimated, even when rejecting the hypothesis that a particular year-effect is zero at the 0.05 level. Thus, for issues of statistical inference it is preferable to group the year-effects. Three sets of years are considered: 2010-2012, 2013-2016 and 2017-20. The models take the following form:

$$\ln\left(\frac{p_{itk}}{1-p_{itk}}\right) = \alpha_{0k} + \alpha_{1k}age + \alpha_{2k}age^2 + \ \delta_{1k}year13\_16 + \delta_{2k}year17\_20$$

where $year13\_16$ is a binary variable which is unity if the birth occurred between 2013-16, and zero otherwise; similarly, $year17\_20$ equals unity if the birth occurred between 2017-20 and zero otherwise. For instance, for first births (parity 0), the parameter estimates (SE) are: $\alpha_{00} = -15.156\ (0.526), \alpha_{10} = 0.844\ (0.035)$, $\alpha_{20} = -0.0133\ (0.0006)$, $\delta_{10} = -0.323\ (0.058)$ and $\delta_{20} = -0.471\ (0.067)$.[8]

Using the estimated model, the estimated parity progression ratios and TFR (and their standard errors) for each time-period are shown in Table 2. As explained in Appendix A, the estimate of the SE of the TFR is based on the 'delta method' using bootstrapped SE's and covariances for the individual PPR's. It appears safe to conclude that the reduction in the TFR between 2010-12 to 2017-20 of 0.41 is not entirely due to sampling variation.

[8] The other parameter estimates are as follows:

	Second births		Third and higher	
	parameter	SE	parameter	SE
Age	0.617	0.044	0.435	0.068
Age sq.	-0.011	0.001	-0.009	0.001
year13_16	0.069	0.059	-0.105	0.069
Year17_20	-0.083	0.075	-0.043	0.083
_cons	-10.374	0.705	-7.249	1.106

Table 2: Simulated PPR's and TFR (bootstrapped SE[a] in parentheses)

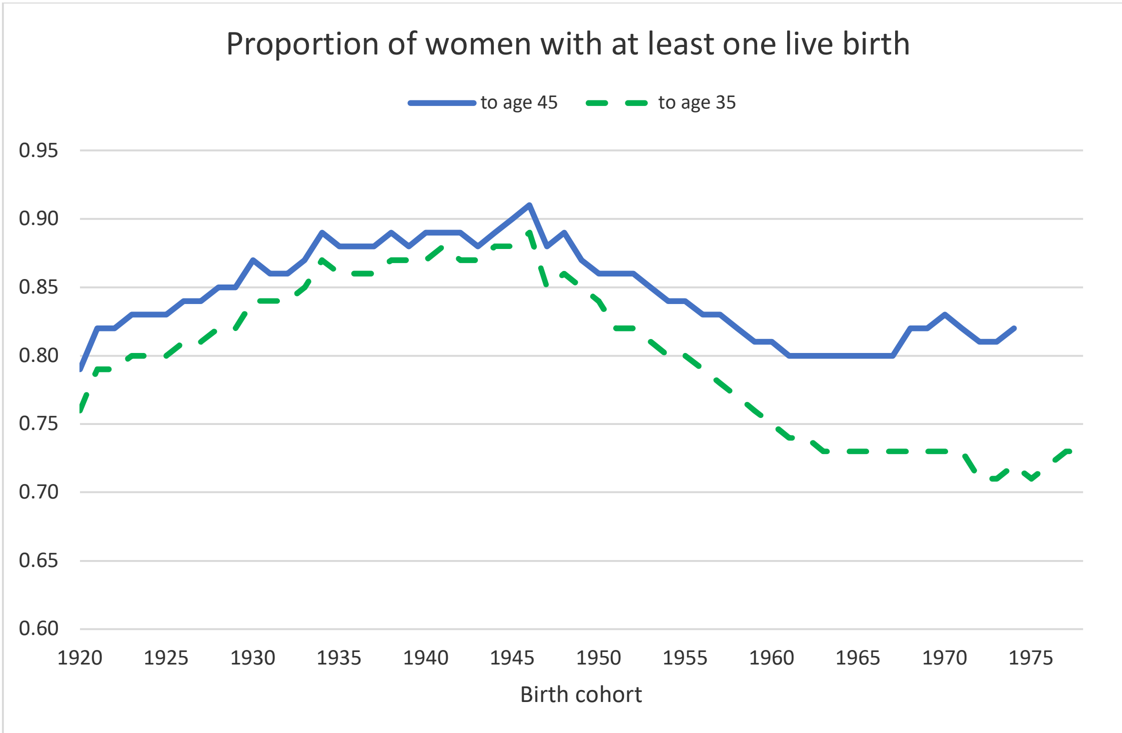
PPR by birth order	2010-12	2013-16	2017-20	Counterfactual[b]
1	0.910 (0.009)	0.830 (0.012)	0.785 (0.017)	0.785 (0.017)
2	0.795 (0.014)	0.776 (0.012)	0.724 (0.018)	0.745 (0.014)
3	0.376 (0.014)	0.333 (0.012)	0.331 (0.016)	0.345 (0.014)
4	0.326 (0.013)	0.292 (0.011)	0.296 (0.015)	0.307 (0.012)
5	0.290 (0.013)	0.262 (0.011)	0.255 (0.015)	0.279 (0.012)
TFR[c]	2.02 (0.040)	1.77 (0.037)	1.61 (0.050)	1.65 (0.048)
Actual TFR, Eng &Wales	1.93	1.83	1.68	

[a]1,000 replications
[b]Assuming $year\_1720 = 1$ for first birth rates only; other age-specific birth rates unaffected.
[c]TFR is the mean TFR over 1,000 replications; the estimate of SE uses the delta method with bootstrapped SE's and covariances for the PPR's (see Appendix A).

The fall in the first birth rate since 2010-12 indicated by the model is consistent with the sharp declines in the birth rates for woman aged under 30 since 2016 (see Figure 3).[9] If the decline in the TFR is almost entirely driven by the fall in the first birth rate, as the counterfactual simulation of the model in the last column of Table 2 suggests, then a recovery in the TFR appears likely unless a much larger proportion of women remain childless than in the past. The age profile for first births with the 2017-20 period effect operating throughout yields a median age at motherhood of 32 instead of 29 (for 2010-12), and PPR_1 indicates that 21% (SE=1.7%) would remain childless, which would take us back to the levels experienced for women born in the early 1920s (Figure 7).

[9] For example, if the age-specific rates under 30 would have remained constant at 2016 values, then the TFR would have only fallen to 1.74 in 2020, rather than 1.60.

Figure 7: Proportion of women having at least one child by birth cohort



3. *Differential fertility decline*

There have been long-standing differences in fertility by a woman's educational attainment, both in timing and completed family size. Further insights into the recent fertility decline were obtained by splitting the sample into two education groups: whether a woman had a university degree (or equivalent) or not by their last interview in the panel.[10] To establish a rough baseline, *Understanding Society* was used to estimate the difference in number of natural children in the household among women born in the 1970s (average age 43.6), who have virtually completed their childbearing. It will underestimate completed fertility because some children may have left home already, some may be living with their father, some may

[10] Similar estimates were made for ethnic groups. Unfortunately, the imprecision of the estimates for the ethnic groups are too large to justify firm conclusions about their differences relative to Whites, although they appear to be smaller than in earlier generations.

have died and some women may still have another child.[11] About one-half of women in these cohorts had a degree. Women without a degree had an average number of children of 1.87 compared with 1.65 for women with a degree.

Separate models were estimated for degree and non-degree women, thereby allowing for different age profiles at each parity for the two groups of women. Using the same method as earlier, the estimates of parity progression ratios and the TFR are shown in Table 3.[12]

[11] In a regression controlling for age at last interview (among women aged at least 40 and born since 1970), the age coefficient is negative (-0.034 SE=0.008), and the predicted average number of children is also 0.22 lower for degree women than non-degree women.

[12] In the data used to the estimate the parity-specific models about 45% of women had a degree. Note that the model disaggregated by woman's education (Table 3) need not produce the same average TFR as the aggregated model summarised in Table 2 because of different age profiles by women's education.

Table 3: Simulated Period PPR's and TFR by Woman's Education
(bootstrapped SE[a] in parentheses)

PPR	(1) 2010-12 No degree	(2) 2017-20 No degree	(3) 2010-12 Degree	(4) 2017-20 Degree	2010-12 Ed. Diff.[c] (3)-(1)	2017-20 Ed. Diff.[c] (4)-(2)
1	0.905 (0.013)	0.742 (0.032)	0.896 (0.013)	0.797 (0.022)	-0.009 (0.019)	0.055 (0.039)
2	0.765 (0.022)	0.646 (0.031)	0.785 (0.020)	0.745 (0.023)	0.021 (0.030)	0.098 (0.039)
3	0.408 (0.017)	0.346 (0.021)	0.290 (0.023)	0.254 (0.022)	-0.118 (0.029)	-0.092 (0.030)
4	0.351 (0.015)	0.306 (0.019)	0.225 (0.018)	0.201 (0.019)	-0.126 (0.024)	-0.105 (0.027)
5	0.315 (0.015)	0.282 (0.019)	0.186 (0.017)	0.169 (0.019)	-0.128 (0.023)	-0.113 (0.027)
TFR[b]	2.01 (0.058)	1.45 (0.081)	1.85 (0.053)	1.57 (0.053)	-0.152 (0.078)	0.126 (0.100)

[a]1,000 replications
[b]TFR is the mean TFR over 1,000 replications; the estimate of SE uses the delta method with bootstrapped SE's and covariances for the PPR's (see Appendix A).
[c]SE is the standard error of the difference.

The most persistent and substantial fertility differences between women by education level are the lower PPRs for third and higher order births among degree educated women. In 2010-12 that produced a lower TFR among them. But there was a larger fertility decline among non-degree women in the subsequent period up to 2017-20, as documented in Table 4, eliminating any meaningful difference in the TFR. The large decline in the TFR was driven by substantial declines in the PPR for the first three birth orders among lower educated women and by a large decline in the PPR for first births among women with a degree.

Table 4: Changes in PPR's and TFR by Women's Education, 2010-12 to 2017-20 (SE[a] in parentheses)

PPR	No degree	Degree	Diff in Change No deg. vs deg.
1	-0.163 (0.034)	-0.100 (0.026)	-0.064 (0.043)
2	-0.118 (0.038)	-0.040 (0.031)	-0.077 (0.049)
3	-0.061 (0.027)	-0.036 (0.032)	-0.025 (0.042)
4	-0.045 (0.024)	-0.024 (0.027)	-0.021 (0.036)
5	-0.033 (0.025)	-0.018 (0.026)	-0.015 (0.036)
TFR[b]	-0.559 (0.099)	-0.281 (0.080)	-0.278 (0.127)

[a] SE is the standard error of the difference.

4. *Conclusions*

Four conclusions concerning recent fertility developments emerge from the analysis in the paper. First, whatever is driving the decline in first birth rates appears to be primarily responsible for the decline in the TFR during the past decade. Second, either Britain is embarking on a regime with levels of childlessness not observed since women born in the 1920s, or we can expect a recovery in period fertility in the future. Third, the analysis indicated a larger decline in fertility among women without a university degree than among degree-educated women, suggesting a compression of educational differentials. Fourth, the study illustrated the value of cross-validation of a model estimated on individual data with external sources and of translating parameter estimates into more easily interpretable concepts such as period parity progression ratios and the total fertility rate.

5. *Acknowledgements*

The work was funded by a Leverhulme Trust Grant for the Leverhulme Centre for Demographic Science. I am also grateful for comments from Richard Breen, Christiaan

Monden and members of the Demography reading group at Oxford for the helpful comments on earlier drafts of the paper.

References

Heckman, J.J. and J. Walker (1990) The relationship between wages and income and the timing and spacing of births: evidence from Swedish longitudinal data. *Econometrica* 58(6): 1411-1441.

Henry, L. (1953) *Fertility of Marriage: a New Method of Measurement* (Paris, 1953; English translation, 1980, UN/ESCAP Population Studies Translation Series No. 3).

Feeney, G. (1983) Population dynamics based on birth intervals and parity progression. *Population Studies* 37:75-89.

Ljung, G. M. and G. E. P. Box (1978). On a Measure of a Lack of Fit in Time Series Models. *Biometrika*. **65** (2): 297–303. doi:10.1093/biomet/65.2.297

Ni Bhrolchain, Maire (1987) Period Parity Progression Ratios and Birth Intervals in England and Wales, 1941-1971: A Synthetic Life Table Analysis. *Population Studies* 41(1):103-125.

Appendix A

PPR SE: Calculating the SE of the Parity Progression Ratio and the TFR

The calculations are simplest for the parity progression ratio for the first birth:

$$PPR_1 = 1 - \prod_{t=0}^{T}(1 - h_{1t})$$

where h_{1t} is the first birth hazard at age t.

If the estimated h_{1t} were independent, the 'delta method' would yield[13]

$$SE(PPR_1) = \sqrt{\sum_t \left(\frac{\partial PPR_1}{\partial h_{1t}}\right)^2 \sigma_{1t}^2} = \sqrt{\sum_t \left(\frac{1-PPR_1}{1-h_{1t}}\right)^2 \sigma_{1t}^2}$$

where σ_{1t} is the standard error of the estimate of h_{1t}. For instance, we might have $\ln\left(\frac{h_{1t}}{1-h_{1t}}\right) = \beta_0 + \beta_1 t + \beta_2 t^2$ where the beta's are estimated from the data. Then σ_{1t} depends on the estimates of the beta's and their covariance matrix.

Because the same age parameter estimates are used in the calculation of each h_{1t}, the estimated h_{1t} are correlated positively across age, and $SE(PPR_1)$ will exceed that calculated by the delta method with independent estimated h_{1t}. An estimate of $SE(PPR_1)$ was, however, obtained using bootstrapping.

Calculation of the SE for higher order PPR's is complicated by the fact that their PPR depends on the inflows into the population at risk from the previous birth order, which is a function of the parameters from the previous birth rate equation, as well as the parameters of the birth hazard for the particular birth order. More specifically, consider the PPR for the second birth. Define h_{2t} as the second birth hazard at age t, and b_{2t} be

---

[13] Because $\frac{\partial PPR_1}{\partial h_{1k}} = \prod_{t\neq k}^{T}(1 - h_{1t}) = \frac{1-PPR_1}{1-h_{1k}}$.

the number of second births at age *t*. Normalise age so that the first age at risk for the first birth (say, 16) is one. There can be no second births at age 1, but from age 2 forwards, second births at age *t* are given by:

$$b_{2t} = h_{2t}\left[b_{1t-1} + \sum_{k=1}^{t-1} b_{1t-1-k} \prod_{j=1}^{k}(1 - h_{2t-j})\right], 2 \leq t \leq 29, b_{10} = 0$$

where b_{1t} are first births at age *t*. Although this equation looks complicated it amounts to saying that the population at risk for a second birth at t (to which h_{2t} is applied) is the population at risk at *t*-1 plus first births at *t*-1 minus second births at *t*-1.

Let B_2 be the sum of b_{2t} over all ages for a given cohort of women (or a synthetic one) and let B_1 be the sum of b_{1t} over all ages. Then the PPR for the second birth is $PPR_2 = \frac{B_2}{B_1}$. It is clear from these expressions that PPR_2 depends on the parameters of h_{1t} as well as h_{2t}. This implies that bootstrapping for the estimate of $SE(PPR_2)$ must re-estimate parameters for the first and second order birth processes at each iteration. As the birth order increases, $SE(PPR_j)$ depends on more previous birth process parameters, and so bootstrapping must iterate over all previous birth processes. (STATA program to be provided with paper.)

Standard errors for the TFR

Letting PPR_j be the progression ratio for birth order *j*, the total fertility rate (or completed family size) is

$$TFR = PPR_1 + PPR_1PPR_2 + PPR_1PPR_2PPR_3 + PPR_1PPR_2PPR_3PPR_4 + \cdots$$

The SE of the TFR is obtained using the delta method. Specifically, let $f_j = \frac{\partial TFR}{\partial PPR_j}$ and let σ_{ij} be the covariance between the estimates of PPR_i and PPR_j, and σ_j is the standard error of the estimate of PPR_j. Then

$$var(TFR) = \sum_j (f_j)^2 \sigma_j^2 + \sum_{i \neq j} f_i f_j \sigma_{ij} \text{ and } SE(TFR) = \sqrt{var(TFR)}.$$

The f_j are taken to be their mean value in the bootstrapped data, consisting of the 1,000 replications.[14] The partial derivatives f_j are:

$$f_1 = \frac{\partial TFR}{\partial PPR_1} = 1 + PPR_2 + PPR_2PPR_3 + PPR_2PPR_3PPR_4 + \cdots$$

$$f_2 = \frac{\partial TFR}{\partial PPR_2} = PPR_1 + PPR_1PPR_3 + PPR_1PPR_3PPR_4 + \cdots$$

$$f_3 = \frac{\partial TFR}{\partial PPR_3} = PPR_1PPR_2 + PPR_1PPR_2PPR_4 + \cdots$$

$$f_4 = \frac{\partial TFR}{\partial PPR_4} = PPR_1PPR_2PPR_3 + \cdots$$

etc. From this series we see that the weights for the SE's applying to σ_j^2 in the estimate of the variance of the TFR estimate in the formula (i.e. ${f_j}^2$) decline rapidly with the order of the birth, as Table A1 illustrates for the model in Table 2 of the text. The intuition here is that errors in estimating the lower order birth rates compound in estimating the TFR, in contrast to the errors for higher order birth rates. On the other hand, σ_j increases as a proportion of PPR_j with birth order because it depends on the estimates of parameters for all previous orders, but σ_j itself does not necessarily increase with birth order, as Table A2 illustrates. The resulting SE estimates for the TFR are shown in the bottom row of Table A2.

[14] Alternatives would be using the mean PPR_j to calculate the f_j or the mean $f_i f_j$ in the delta method formula.

Table A1: Weights on PPR SE's $\left(\frac{\partial TFR}{\partial PPR_j}\right)^2$ for *SE(TFR)* calculations

Birth order	2010-12	2013-16	2017-20
1	4.93	4.54	4.22
2	1.95	1.46	1.30
3	1.06	0.78	0.61
4	0.12	0.07	0.06
5	0.01	0.00	0.00

Table A2: Bootstrapped SE's from Table 2 of text

Birth order	2010-12	2013-16	2017-20
1	0.009	0.012	0.017
2	0.014	0.012	0.018
3	0.014	0.012	0.016
4	0.013	0.011	0.015
5	0.013	0.011	0.015
TFR*	**0.040**	**0.037**	**0.050**

*From the delta method.

Appendix B

Table B.1 Descriptive Statistics

Continuous or dichotomous variables

Variable	Obs	Mean	Std. Dev.	Min	Max
Birth	71,078	0.065	0.246	0	1
Age	71,078	31.883	8.289	16	49
Scotland	71,078	0.079	0.271	0	1
Interview year	71,074	2014.530	2.628	2010	2020
duration (parity>=1)	37,899	4.875	4.060	0	15

Categorical variables (proportions)

Education	
No qualifications	0.042
Low	0.249
Medium	0.266
Degree plus	0.443
Ethnic origin	
White (British and other)	0.750
Mixed: White & BAME	0.029
Indian	0.046

Pakistani	0.053
Bangladeshi	0.034
Chinese+Other Asian	0.023
Carribean	0.018
African & other black	0.039
Other ethnic minority	0.010